

47. (a) We use the Bernoulli equation: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$, where h_1 is the height of the water in the tank, p_1 is the pressure there, and v_1 is the speed of the water there; h_2 is the altitude of the hole, p_2 is the pressure there, and v_2 is the speed of the water there. ρ is the density of water. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then becomes $\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$ and

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2(9.8 \text{ m/s}^2)(0.30 \text{ m})} = 2.42 \text{ m/s} .$$

The flow rate is $A_2 v_2 = (6.5 \times 10^{-4} \text{ m}^2)(2.42 \text{ m/s}) = 1.6 \times 10^{-3} \text{ m}^3/\text{s}$.

- (b) We use the equation of continuity: $A_2 v_2 = A_3 v_3$, where $A_3 = \frac{1}{2}A_2$ and v_3 is the water speed where the area of the stream is half its area at the hole. Thus $v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84 \text{ m/s}$. The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s . Since the pressure is the same throughout the fall, $\frac{1}{2}\rho v_2^2 + \rho g h_2 = \frac{1}{2}\rho v_3^2 + \rho g h_3$. Thus

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{(4.84 \text{ m/s})^2 - (2.42 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 0.90 \text{ m} .$$